

# Expected Number Counts of Radio Galaxy Clusters

Wei Zhou \*

National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012

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**Abstract** Clusters of galaxies may contain radio sources if they still experience successive mergers at present. This has been confirmed by radio observations that about 30% of nearby clusters possess radio halos. We present a theoretical prediction of radio cluster counts using a semi-analytic approach which incorporates the empirical correlation between radio power and dynamical mass of clusters, and the cluster mass function described by the Press-Schechter formalism. The total population of radio clusters over the whole sky and their redshift distribution are given.

**Key words:** cosmology: theory — galaxies: cluster: general — radio sources: extended

## 1 INTRODUCTION

Over the past years, diffuse radio halos have been detected in a few tens of nearby, rich clusters. They often extend to a distance of  $\sim 1$  Mpc from the cluster centers, and have regular shape, low surface brightness and steep radio spectrum. Some clusters also contain peculiar radio structures called radio relics. Both radio halos and relics are believed to arise from the merging of sub-cluster structures (Buote 2001).

The first radio halo, Coma C, was detected about 30 years ago, and the number of radio sources has increased recently thanks to the NRAO VLA Sky Survey (Condon et al. 1998). About 20 clusters have been detected to contain radio halos. Among these, Liang et al. (2000) found that the diffuse radio source in the known hottest cluster, 1E 0657–56, has a morphology similar to the X-ray emission from the hot intracluster medium. In particular, they have shown that there is a tight correlation between the radio power  $P_{1.4\text{GHz}}$  (at 1.4 GHz rest frame) and the X-ray temperature ( $T_x$ ) based on ten well-studied radio halos, i.e.,  $P_{1.4\text{GHz}}$  is positively correlated with  $T_x$ . More recently, Govoni et al. (2001) observed six clusters with Deep Very Large Array and compared them with the X-ray data. They confirmed that the radio power is strongly associated with the underlying gravitational potential, manifesting itself by a linear correlation between the radio power and the cluster gravitational mass. Alternatively, Buote (2001) detected a correlation between the 1.4 GHz power of radio halo (or relic) and the mag-

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\* E-mail: zw@class2.bao.ac.cn

nitude of the dipole power ratio ( $P_1/P_0$ ) such that  $P_{1.4\text{ GHz}} \propto P_1/P_0$ . It is noted that radio sources are common in the clusters that are experiencing violent mergers ( $P_1/P_0 \geq 0.5 \times 10^{-4}$ ).

The detection of radio emission in clusters gives support to the hierarchical model of structure formation, in which clusters form by gravitational aggregation of individual low-mass objects and such process should still be observable today. The question we will address in this paper is how many clusters can be seen over the whole sky in radio wavelength. Although a sophisticated estimate of the radio cluster counts may be achieved by combining the extended Press-Schechter (PS) formalism with the merger rates in hierarchical models of galaxy formation, we may arrive at a crude estimate of the radio cluster counts from a much simpler approach. For a given radio flux, we can calculate the radio power according to different redshift and convert it into the corresponding cluster mass using the empirical relation between gravitating mass and radio power (Govoni et al. 2001). The distribution and evolution of clusters are then described by the PS mass function. Finally, the total number of radio clusters can be obtained by integrating the PS mass function over the mass and redshift ranges.

## 2 THEORETICAL PREDICTION

### 2.1 Number Counts

We compute the number of radio clusters with radio flux greater than  $S_{1.4\text{ GHz}}$  by

$$N(> S_{1.4\text{ GHz}}) = \int_0^\infty \left[ \int_{M(S_{1.4\text{ GHz}})}^\infty \frac{dn}{dM} dM \right] \frac{dV}{dz} dz, \quad (1)$$

where  $M(S_{1.4\text{ GHz}})$  is the lowest cluster mass whose radio halo can be seen above the radio flux limit at 1.4 GHz, and  $dn/dM$  is the comoving number density of virialized clusters of mass in the range of  $M \sim M + dM$  at redshift  $z$ . We have

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \frac{\rho_m}{M} \frac{d \ln \sigma(M)}{dM} \nu_c e^{-\frac{\nu_c^2}{2}}, \quad (2)$$

where  $\rho_m$  is the present average comoving matter density of the universe,  $\nu_c = \delta_{\text{crit}}(z)/\sigma(M)$  is the threshold function for collapse, and  $\sigma(M)$  is the present mass standard deviation for the fluctuation spectrum fitted on mass  $M$ . We have

$$\sigma^2(M) = \frac{1}{2\pi^2} \int_0^\infty k^2 p(k) W^2(rk) dk, \quad (3)$$

where  $W(x) = 3(\sin x - x \cos x)/x^3$  works as the Fourier-Space representation of a spherical top-hat window function. We parameterize the power spectrum of initial fluctuation by  $p(k) = Ak^n T^2(k)$ . The normalization of the power spectrum is realized by COBE four years data and the primordial spectrum is adopted to have  $n = 1$ . We take the transfer function  $T^2(k)$  given by Bardeen et al. (1986) for an adiabatic CDM model. Another important parameter in equation (2) is the critical density of collapse,  $\delta_{\text{crit}}(z)$ . The overdensity linearly extrapolated to the present epoch is characterized by the normalized linear growth factor  $D(z)$  and that is  $\delta_{\text{crit}}(z) = \delta_{\text{crit}}(0)/D(z)$ , where  $\delta_{\text{crit}}(0) = 1.68$  represents the critical density at redshift  $z = 0$  and has a weak dependence on cosmological models. It is convenient to express the growth factor as  $D(z) = [g(z)/g(0)]/(1+z)$ , where the best-fit  $g(z)$  has been provided by Carroll, Press and Turner (1992) as

$$g(z) = \frac{(5/2)\Omega_m(z)}{\Omega_m(z)^{4/7} - \Omega_\Lambda(z) + (1 + \Omega_m(z)/2)(1 + \Omega_\Lambda(z)/70)}, \quad (4)$$

where  $\Omega_m(z) = \Omega_m(1+z)^3/E^2(z)$ ,  $\Omega_\Lambda(z) = \Omega_\Lambda/E^2(z)$ , and  $E^2(z) = \Omega_m(1+z)^3 + \Omega_k(1+z) + \Omega_\Lambda$ .

## 2.2 The Relationship between Radio Flux and Gravitational Mass

There are two essential problems for radio clusters. One concerns the mechanism that accelerates the relativistic electrons to produce non-thermal radio emission. Another is the question of rarity of radio clusters (Buote 2001). It is believed that mergers should be able to provide enough energy ( $\sim 10^{64}$  erg) and account for the formation of the radio emission. However, the underlying gravitational mass of a cluster is probably the key factor since the energy available for accelerating relativistic particles during a merger scales as  $\sim M^2$  (Buote 2001). This is justified by a direct correlation between the radio power and the gravitating mass shown by Govoni et al. (2001), see below. Indirect support to the argument comes from the correlations between X-ray luminosity/temperature and radio power.

In Figure 1, we plot the radio power calculated at 1.4 GHz versus the virial mass, together with the best-fit power-law  $P_{1.4\text{GHz}} = 10^a M^b$ . The data are taken from Govoni et al. (2001), but we have converted the mass within 3 Mpc into the value within the virial radius by using

$$r_{\Delta_c}(T_x) = r_{10}(\Delta_c) \left( \frac{T_x}{10 \text{ keV}} \right)^{1/2}, \quad (5)$$

where the normalization  $r_{10}(\Delta_c)$  is the virial radius of a 10 keV cluster at density contrast  $\Delta_c$ , and the cluster mass is assumed to be proportional to  $r$  when the radius greatly exceeds the core radius. It turns out that the best-fit relation is  $a = -4.9$  and  $b = 1.9$  when the fit is weighted by a factor of  $\sigma^{-2}$ , where  $\sigma$  represents the measurement error.

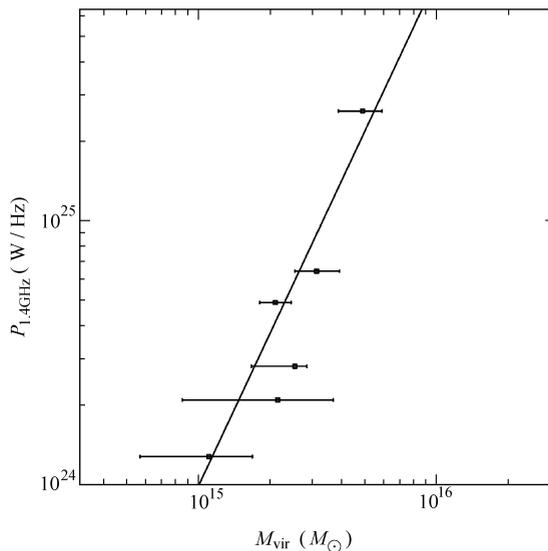


Fig. 1 Relationship between the halo power at 1.4 GHz ( $P_{1.4\text{GHz}}$ ) and the virial mass for clusters (Govoni et al. 2001). The best-fit power-law is  $P_{1.4\text{GHz}} = 10^a M^b$ , where  $a = -4.9$  and  $b = 1.9$ .

The cluster radio flux is related to the radio power by  $P_{1.4\text{ GHz}} = 4\pi D_L^2 S_{1.4\text{ GHz}}$ , where the luminosity distance  $D_L$  at redshift  $z$  can be explicitly given, by

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{(1+z)\sqrt{1 + \Omega_m z + \Omega_\Lambda [1 - (1+z)^{-2}]}} \quad (6)$$

for a cosmological model with  $\Omega_k = 0$ . Thus the relation between radio flux and cluster mass can be obtained.

In the actual calculation, about 30% clusters are assumed to have radio emission (Boute 2001), and such a fraction is extrapolated to high redshift  $\sim 1$ . This is a rough assumption, which we will discuss in Section 3.

### 2.3 Results

We take the  $\Lambda$ CDM model ( $\Omega_m = 0.3, \Omega_\Lambda = 0.7$  and  $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) for our numerical calculation. In Figure 2, we plot the total number of the expected radio clusters with radio flux larger than a given value  $S_{1.4\text{ GHz}}$ . It shows that the total number decreases with increasing radio flux, as naturally expected. For example, the total number reaches  $\sim 10^{-1}$  per unit solid angle at 0.01 Jy while it reduces to  $\sim 10^{-3}$  per unit solid angle at 1.0 Jy. Over  $S_{1.4\text{ GHz}} \sim 20 \text{ Jy}$ , there would be no radio clusters over the whole sky. This is consistent with existing observations,  $S_{1.4\text{ GHz}} < 1.0 \text{ Jy}$  (Giovannini et al. 1999; Giovannini et al. 2000; Liang et al. 2000).

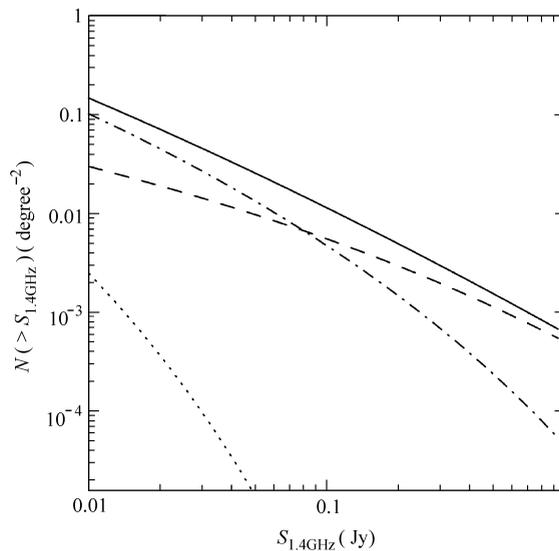


Fig. 2 Radio cluster number ( $N(> S_{1.4\text{ GHz}})$ ) versus the halo radio flux at 1.4 GHz ( $S_{1.4\text{ GHz}}$ ). The solid line represents the total number. The contributions from redshift range 0 – 0.1, 0.1 – 0.5, 0.5 – 1.0 are shown by the long-dashed line, dot-dashed line and dotted line, respectively.

We also demonstrate the dependence of the predicted radio clusters on redshift in Figure 3. Most of the radio clusters on the sky are located at lower redshifts. The number is peaked at

$z = 0.14$ ,  $z = 0.06$  and  $z = 0.04$  for three different given radio flux limit respectively. Due to the cosmic evolution, clusters come into being at lower redshift and in observation, because of selection effects, we cannot see an object too far away or too near us. Therefore the peak appears at a redshift in a certain range.

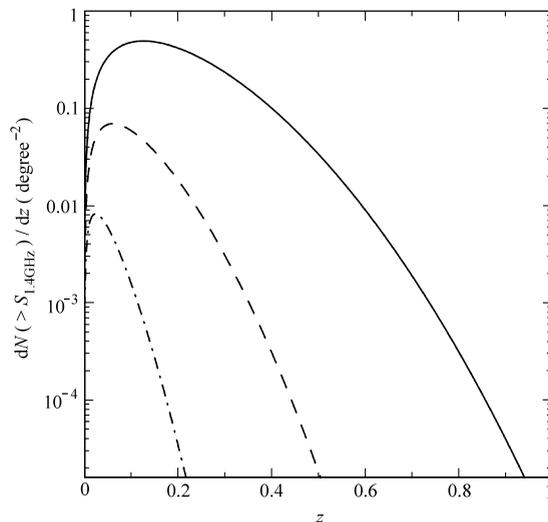


Fig. 3 Radio cluster number ( $dN(> S_{1.4\text{GHz}})/dz$ ) versus redshift ( $z$ ). The numbers with different radio flux limit at 1.4 GHz ( $S_{1.4\text{GHz}}$ ) are shown as solid line (0.01 Jy), long dashed line (0.1 Jy) and dot-dashed line (1.0 Jy), respectively.

### 3 DISCUSSION

Our crude estimate of radio cluster counts is useful for future detection of clusters in radio wavelength, however, there are several shortcomings which need to be discussed. First, the current sample of radio clusters is still small, as a result the empirical relation based on such a small number statistics may contain large uncertainty. The predicted number is sensitive to the lower limit  $M(S_{1.4\text{GHz}})$ . A larger sample of radio clusters is needed to tighten the  $M-S_{1.4\text{GHz}}$  relation. Secondly, we did not consider the merger history of clusters, which is reflected by a constant fraction of radio clusters used in our numerical estimate. This fraction is only valid at low redshift, and will be uncertain at high redshift where the merger rate should be re-evaluated by either theoretical considerations or new radio observations. For the latter, it turns out to be extremely difficult with current technique. In this regard, our extrapolation of both the radio cluster fraction and the  $M-S_{1.4\text{GHz}}$  relation to high redshift is tentative and future result of the redshift evolution of the fraction may improve the calculations.

Nevertheless, our estimate of the radio cluster counts based on current knowledge of radio observations and analytic models of cluster distribution serves as a useful guide for further investigations of the issue.

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