

## A Toy Model for 3:2 Ratio of kHz QPO Frequency in Black Hole X-ray Binaries

Ding-Xiong Wang<sup>\*</sup>, Yong-Chun Ye, Ren-Yi Ma and Xiao-Long Gong

Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, China

**Abstract** Production of pairs of high frequency quasi-periodic oscillations (QPOs) in black hole X-ray binaries is discussed based on a model of non-axisymmetric magnetic coupling of a rotating black hole (BH) with its surrounding accretion disk, in which a puzzling 3:2 ratio of the upper frequency to the lower frequency is explained. In addition, the correlation of the pairs of high frequency QPOs with the jets from microquasars is discussed.

**Key words:** accretion – accretion disks – black hole physics – X-rays: stars – Microquasars

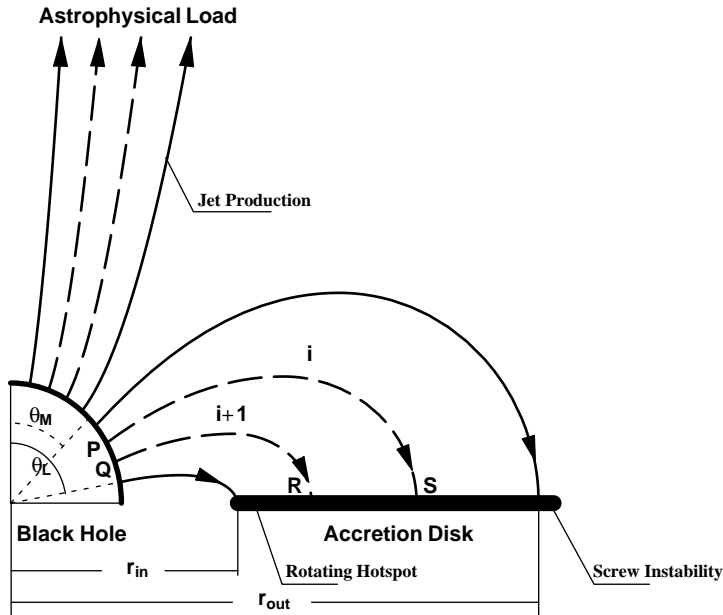
### 1 INTRODUCTION

Quasi-periodic oscillations (QPOs) in X-ray binaries have become a very active research field since the launch of the RXTE. A key feature in these sources is that some of high frequency QPOs appear in pairs. There are now five black hole (BH) X-ray binaries that exhibit transient high frequency QPOs, of which three sources have pairs occurring in GRO J1655–40 (450, 300 Hz), GRS 1915+105 (168, 113 Hz), and XTE J1550–564 (276, 184 Hz) with a puzzling 3:2 ratio of the upper frequency to the lower frequency.

A number of different mechanisms have been proposed to explain the origin of high frequency QPO pairs in BH X-ray binaries (Wagoner et al. 2001; Strohmayer 2001; Abramowicz & Kluzniak 2001). It seems that more than one physical model is required to fit all of the high frequency QPO observations. In this paper we propose a model to explain high frequency QPO pairs in BH X-ray binaries, which is based on non-axisymmetric magnetic coupling (MC) of a rotating BH with its surrounding accretion disk. It turns out that the upper frequency arises from a rotating hotspot near the inner edge of the disk, and the lower frequency might be produced by the screw instability at somewhere away from the inner edge. Calculations show that the high frequency QPO pairs in BH X-ray binaries, GRO J1655–40 (450, 300 Hz), GRS 1915+105 (168, 113 Hz) and XTE J1550–564 (276, 184 Hz), are well fitted to the observations in some value ranges of these parameters. It is argued that these high frequency QPO pairs produced in our model might accompany jets production.

---

<sup>\*</sup> E-mail: dxwang@mail.hust.edu.cn



**Fig. 1** Poloidal magnetic field connecting a rotating BH with a remote astrophysical load and the surrounding disk.

## 2 DESCRIPTION OF MODEL

In order to discuss high frequency QPO pairs in the context of BH magnetosphere we make the following assumptions.

(1) The configuration of poloidal magnetic field in BH magnetosphere is depicted in Figure 1, where  $r_{\text{in}}$  and  $r_{\text{out}}$  are the radii of the inner and outer boundaries of the MC region, respectively. The angle  $\theta_M$  indicates the angular boundary between the open and closed field lines on the horizon.

(2) The toroidal profile of the poloidal magnetic field on the BH horizon is assumed to be non-axisymmetric, and the profile function  $f(\phi)$  is expressed by (Wang et al. 2003a, hereafter W03)

$$B_H^p(\phi) = \sqrt{\langle (B_H^p)^2 \rangle} f(\phi), \quad f(\phi) \equiv \begin{cases} 1 + \delta, & 0 < \phi < \Delta\phi, \\ 1, & \Delta\phi \leq \phi \leq 2\pi, \end{cases} \quad (1)$$

where  $\sqrt{\langle (B_H^p)^2 \rangle}$  is root-mean-square of the poloidal magnetic field over the angular coordinate from  $\theta = 0$  to  $\theta_L$ .

(3) The magnetosphere is assumed to be force-free outside the BH and the disk, and the closed magnetic field lines are frozen in the disk. The disk is thin and perfectly conducting, lies in the equatorial plane of the BH with the inner boundary being at the marginally stable orbit.

(4) The poloidal magnetic field remains unchanged on the horizon, and it varies as a power law with the radial coordinate of the disk:  $B_D^p \propto \xi^{-n}$ , where  $B_D^p$  is the poloidal magnetic field on the disk. The parameter  $n$  is the power-law index, and  $\xi \equiv r/r_{ms}$  is the radial coordinate

on the disk, which is defined in terms of the radius  $r_{ms}$  of the marginally stable orbit (Novikov & Thorne 1973).

(5) The magnetic flux connecting the BH with its surrounding disk takes precedence over that connecting the BH with the remote load.

Assumption 5 is crucial for the coexistence of the BZ and MC processes (Wang et al. 2003b). The BZ and MC powers in non-axisymmetric case are expressed by

$$P_{\text{BZ}}^{NA} = \lambda P_{\text{BZ}}^A, \quad (2)$$

$$P_{\text{MC}}^{NA} = \lambda P_{\text{MC}}^A, \quad (3)$$

where the powers  $P_{\text{BZ}}^A$  and  $P_{\text{MC}}^A$  in axisymmetric case have been derived in our previous work (Wang et al. 2002, hereafter W02). The parameter  $\lambda$  is expressed in terms of  $\delta$  and  $\varepsilon \equiv \Delta\phi/2\pi$  by

$$\lambda = [(1 + \delta)\varepsilon + (1 - \varepsilon)]^2 = (1 + \delta\varepsilon)^2. \quad (4)$$

### 3 PRODUCTION OF HIGH FREQUENCY QPO PAIRS

Very recently, we discussed the screw instability of the magnetic field in axisymmetric magnetosphere, and found that the coexistence of the BZ and MC processes always accompanies the screw instability (Wang et al. 2004, hereafter W04). In W03 we argued that the rotating hotspot contributes QPO frequency  $\nu_{\text{HS}}$  expressed by

$$\nu_{\text{HS}} = \nu_0 (\xi_{\text{HS}}^{3/2} \chi_{ms}^3 + a_*)^{-1}, \quad (5)$$

where  $\nu_0 \equiv (m_{\text{BH}})^{-1} \times 3.23 \times 10^4 \text{ Hz}$ . The parameter  $\xi_{\text{HS}}$  corresponds to the maximum of function  $F_{\text{QPO}}$ , which is expressed by

$$F_{\text{QPO}} \equiv r^2 F / r_{ms}^2 F_0 = \xi^2 F_{\text{MC}} / F_0. \quad (6)$$

The involved function  $F_{\text{QPO}}$  is given in W03.

Now we devoted the rest of this section to the correlation of the screw instability with the lower frequency of high frequency QPO pairs.

Gruzinov (1999) argued that a Kerr BH, being connected with a disk by a bunch of closed field lines, can flare quasi-periodically due to the screw instability of the magnetic field. In W04 we suggested that the criterion for the screw instability in the MC process could be expressed by

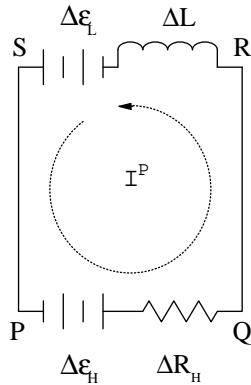
$$(2\pi\varpi_D/L) B_D^p / B_D^T < 1, \quad (7)$$

where  $B_D^p$  and  $B_D^T$  are the poloidal and toroidal components of the magnetic field on the disk, respectively, and  $\varpi_D$  is the cylindrical radius on the disk. The quantity  $L$  in equation (7) is the poloidal length of the closed field line connecting the BH with the disk. Equation (7) is derived based on the Kruskal-Shafranov criterion (Kadomtsev 1966), which implies that the screw instability will occur, if the magnetic field line turns around itself about once.

From equation (7) we can determine the minimum radial coordinate  $\xi_{SC}$  for the screw instability by the following equation,

$$(2\pi\varpi_D/L) F(a_*; \xi, n) = 1, \quad (8)$$

where  $F(a_*; \xi, n)$  is given in W04. Equation (8) implies that the disk region for the screw instability (henceforth DRSC) is  $\xi_{SC} < \xi < \infty$ . We suggest that the lower QPO frequency



**Fig. 2** One loop of equivalent circuit for screw instability in the BH magnetosphere.

is related to the flares repeating quasi-periodically in DRSC. Considering that the process of releasing magnetic energy is much faster than that of rebuilding magnetic energy, we regard the time lapsed in rebuilding magnetic energy as the time interval between two sequential flares, and the lower QPO frequency can be estimated by  $\nu_{SC} = 1/t_{SC}$ .

Considering the existence of the toroidal magnetic field, we introduce a new component, the inductor, and regard the process of rebuilding magnetic energy in DRSC as a transient process in the circuit with resistor and inductor in series (R-L circuit) as shown in Figure 2.

$$\Delta L = \Delta \Psi^T / I^P, \quad (9)$$

where  $I^P$  and  $\Delta \Psi^T$  are the current flowing in the circuit and the flux of the toroidal magnetic field penetrating through the circuit, respectively.

The flux  $\Delta \Psi^T$  is expressed as  $\Delta \Psi^T = \oint_{\text{loop}} B^T \sqrt{g_{rr}g_{\theta\theta}} dr d\theta$ . The toroidal magnetic field  $B^T$  is proportional to the current  $I^P$ , and the latter is governed by the following equation,

$$\Delta L \frac{dI^P}{dt} + \Delta R_H I^P = \Delta \varepsilon_H + \Delta \varepsilon_L. \quad (10)$$

Considering the initial condition,  $I^P = 0$ , we have the solution:  $I^P(t) = I_{\text{steady}}^P (1 - e^{-\Delta R_H t / \Delta L})$ . Thus the relaxation time in the transient process is characterized by  $\tau = \Delta L / \Delta R_H$ . Therefore we expect that the screw instability might occur again after time  $t_{SC}$ , and the lower QPO frequency is limited by  $\nu_{SC} = (\lambda_\tau \tau)^{-1}$ .

The concerned data for 3:2 ratio of QPO frequency are shown in Table 1. The value ranges of the BH mass corresponding to GRO J1655–40, GRS 1915+105 and XTE J1550–564 are adopted from Greene et al. (2001), McClintock & Remillard (2003) and Orosz et al. (2002), respectively. The quantities  $E_{HS}$  and  $E_{SC}$  are the energy of the rotating hotspot and that released between two sequential flares arising from the screw instability, and they are regarded as effective energy of black body radiation corresponding to the values of  $F_{MC}$  at  $\xi_{HS}$  and  $\xi_{SC}$ , respectively.

It turns out that 3:2 ratio of QPO frequency can be fitted by the parameters  $a_*$  and  $n$  with  $\lambda_\tau = 3$ , while the ratio is not sensitive to the values of the parameters  $\delta$  and  $\varepsilon$ .

**Table 1** Pairs of high frequency QPOs produced by a rotating hotspot and screw instability with  $\delta = 0.5$ ,  $\varepsilon = 0.2$  and  $\lambda_r = 3$ , where MQ-I, MQ-II and MQ-III represent GRO J1655–40, GRS 1915+105 and XTE J1550–564, respectively.

MQ	$a_*$	$n$	$m_{\text{BH}}$	Rotating Hotspot			Screw Instability		
				$\xi_{\text{HS}}$	$E_{\text{HS}}(\text{keV})$	$\nu_{\text{HS}}$	$\xi_{\text{SC}}$	$E_{\text{SC}}(\text{keV})$	$\nu_{\text{SC}}$
I	0.67	5.52	6.8	1.302	$7.06 \times 10^{-3} B_4^{1/2}$	450	2.733	$3.29 \times 10^{-3} B_4^{1/2}$	300
	0.57	5.84	5.8	1.309	$6.03 \times 10^{-3} B_4^{1/2}$	450	2.778	$2.77 \times 10^{-3} B_4^{1/2}$	300
II	0.67	5.52	18	1.302	$7.05 \times 10^{-3} B_4^{1/2}$	168	2.733	$3.28 \times 10^{-3} B_4^{1/2}$	113
	0.34	6.40	10	1.461	$3.46 \times 10^{-3} B_4^{1/2}$	168	2.983	$1.58 \times 10^{-3} B_4^{1/2}$	113
III	0.69	5.43	11.5	1.302	$7.31 \times 10^{-3} B_4^{1/2}$	276	2.733	$3.40 \times 10^{-3} B_4^{1/2}$	184
	0.50	6.01	8.5	1.320	$5.31 \times 10^{-3} B_4^{1/2}$	276	2.851	$2.40 \times 10^{-3} B_4^{1/2}$	184

#### 4 CORRELATION OF HIGH FREQUENCY QPO PAIRS WITH JETS FROM MICROQUASARS

Since GRO J1655–40, GRS 1915+105 and XTE J1550–564 are also microquasars, there must be some correlation between high frequency QPO pairs with jets from these objects. We can obtain this correlation directly. In our model the jet produced by the BZ process is related to the open field lines with the angular region on the horizon from 0 to  $\theta_M$ . So we expect that high frequency QPO pairs accompany the jets from these microquasars.

By using equations (2) we find that the jets driven by the BZ power does accompany high frequency QPO pairs with the values of  $a_*$  and  $n$  listed in Table 2.

**Table 2** The BZ powers accompanying high frequency QPO pairs for  $\delta = 0.5$ ,  $\varepsilon = 0.2$  and  $\lambda_r \approx 3$ , where MQ-I, MQ-II and MQ-III represent GRO J1655–40, GRS 1915+105 and XTE J1550–564, respectively.

MQ	$a_*$	$n$	$m_{\text{BH}}$	$\theta_M$	$P_{\text{BZ}}^{NA}/P_0$	$P_{\text{BZ}}^{NA}(\text{erg s}^{-1})$
I	0.67	5.52	6.8	0.622	$4.62 \times 10^{-3}$	$1.41 \times 10^{28} B_4^2$
	0.57	5.84	5.8	0.619	$3.24 \times 10^{-3}$	$7.18 \times 10^{27} B_4^2$
II	0.67	5.52	18	0.620	$4.51 \times 10^{-3}$	$9.63 \times 10^{28} B_4^2$
	0.34	6.40	10	0.592	$0.99 \times 10^{-3}$	$6.52 \times 10^{27} B_4^2$
III	0.69	5.43	11.5	0.617	$4.74 \times 10^{-3}$	$4.13 \times 10^{28} B_4^2$
	0.50	6.01	8.5	0.607	$2.30 \times 10^{-3}$	$1.09 \times 10^{28} B_4^2$

**Acknowledgements** This work is supported by the National Natural Science Foundation of China under grants 10173004, 10373006 and 10121503.

## References

- Abramowicz M. A., Kluzniak W., 2001, *A&A*, 374, L19  
Greene J., Bailyn C. D., Orosz J. A., 2001, *ApJ*, 554, 1290  
Gruzinov A., 1999, astro-ph/9908101  
Kadomtsev B. B., 1966, *Rev. Plasma Phys.*, 2, 153  
McClintock J. E., Remillard R. A., 2003, astro-ph/0306213  
Novikov I. D., Thorne K. S., 1973, in *Black Holes*, ed. Dewitt C, (Gordon and Breach, New York)  
p.345  
Orosz et al., 2002, *ApJ*, 568, 845  
Strohmayer T. E., 2001a, *ApJ*, 552, L49  
Wagoner R. V., Silbergleit A. S., Ortega-Rodriguez M., 2001, *ApJ*, 559, L25  
Wang D.-X., Xiao K., Lei W.-H., 2002, *MNRAS*, 335, 655 (W02)  
Wang D.-X., Ma R.-Y., Lei W.-H., et al., 2003a, 2003, *MNRAS*, 344, 473 (W03)  
Wang D.-X., Ma R.-Y., Lei W.-H., Yao G.-Z., 2003b, *ApJ*, 595, 109  
Wang D.-X., Ma R.-Y., Lei W.-H., et al., 2004, *ApJ*, 601, 1031 (W04)