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Outflows and Inflows in Astrophysical Systems

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Abstract We seek for self-similar solutions describing the time-dependent evolution of self-gravity systems with either spherical symmetry or axisymmetric disk geometry. By assuming self-similar variable $x \equiv r/at$ where a is isothermal sound speed we find self-similar solutions extending from the initial instant $t = 0$ to the final stage $t \to \infty$ using standard semi-analytical methods. Different types of solutions are constructed, which describe overall expansion or collapse, envelope expansion with core collapse (EECC), the formation of central rotationally supported quasi-equilibrium disk as well as shocks. Though infinitely many, these self-similarity solutions have similar asymptotic behaviors which may impose diagnosis on the velocity and density structures in astrophysical systems.

Key words: hydrodynamics – ISM: clouds – shock waves – stars: formation – winds, outflows

1 INTRODUCTION

Far away from initial and boundary conditions, dynamical evolution of fluid systems may lead to self-similar phases with physical variable profiles shape-invariant and magnitudes properly scaled (Sedov 1959; Landau & Lifshitz 1959). Similarity methods, which transform partial differential equations (PDEs) to ordinary differential equations (ODEs), greatly simplify nonlinear problems. For spherical systems, several well-known self-similar solutions were found to describe the collapse of an isothermal cloud in the context of star formation (Larson 1969; Penston 1969; Shu 1977; Hunter 1977; Whitworth & Summers 1985). The first is the LP solution independently found by Larson (1969) and Pensten (1969); the second is the "expansion wave collapse" solution (EWCS) found by Shu (1977), who also discovered other solutions with central point mass from the initial instant $t = 0$ to the final stage $t \to +\infty$; Hunter (1977) discovered infinitely many discrete analytic solutions within the pre-catastrophic period which share with the LP solution that they are regular at $x \to 0^-$ although some of these solutions can not be continued in the post-catastrophic period from $t = 0$ to $t \to +\infty$. Hunter's results were extended by Whitworth

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& Summers (1985, hereafter WS), who allowed weak discontinuities across the sonic critical line and hence constructed continuous bands instead of Hunter's discrete solutions. But their solutions with weak discontinuities were criticized by Hunter (1986) that are only of mathematical interest and are physically unstable. Recently, Lou & Shen (2004, hereafter LS04) re-examined this classic problem and derived new solutions in the 'semi-complete space' ($0 < t < +\infty$), in contrast to the 'complete space' ($-\infty < t < +\infty$) taken by Hunter (1977) and WS.

The realistic situation may deviate from the spherical collapse. In essence, the presence of rotation or magnetic fields will cause the collapsing system to be more and more flattened (e.g., Nakamura, Hanawa, & Nakano 1995, hereafter NHN) so that the system will later evolve disk-like. Models considering self-similar evolutions in a thin disk have been studied in many papers (e.g., NHN; Li & Shu 1997; Saigo & Hanawa 1998; Krasnopolsky & Königl 2002) with or with out rotation and magnetic fields. Axisymmetry is assumed in the above papers to simplify the problem.

In this work, we plan to establish the whole solution structure for an isothermal fluid system with either spherical symmetry or axisymmetric disk geometry within the period from $t = 0$ to $t \to \infty$. In comparison with the previous solutions found by other authors, we particularly emphasize the existence of such solutions that the innermost part (core) is collapsing towards free-fall while simultaneously the outer part (envelope) is expanding and approaching constant wind. However, the overall collapse region is expanding in a self-similar manner. These envelope expansion with core collapse (EECC) solutions exist in the spherical and non-rotating disk cases but not the rotating disk case (see Section 3.2). Furthermore, as an non-trivial extension we can incorporate purely azimuthal magnetic fields in the disk cases (details are not discussed here).

2 SPHERICAL CASES

The basic formalism for the problem with spherical symmetry is well-established by many authors. By the definition of self-similar variable $x \equiv r/at$ where r, a, t are the radial coordinate, isothermal sound speed and time coordinate respectively, the reduced ordinary differential equations (ODEs) are as follows

$$[(x-v)^2-1]\frac{dv}{dx} = \left[\alpha(x-v) - \frac{2}{x}\right](x-v) \ , \ [(x-v)^2-1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x-v)\right](x-v) \ ,$$
$$m = x^2\alpha(x-v) \ , \tag{1}$$

where $\alpha(x)$, $v(x)$, $m(x)$ are the reduced density, radial bulk velocity and enclosed mass, respectively. The physical density ρ, radial bulk velocity u and enclosed mass M are obtained by the similarity transformation

$$\rho(r,t) = \frac{\alpha(x)}{4\pi G t^2}, \qquad u(r,t) = av(x), \qquad M(r,t) = \frac{a^3 t}{G} m(x). \tag{2}$$

By expanding the solutions in power serials at large and small x; and with careful treatment of the sonic critical line one may construct various types of self-similar solutions which are present in LS04. In addition to the shock-free solutions present in LS04, we can also construct shocked similarity solutions, which are applied to the star-forming regions and H II regions (Shen & Lou 2004).

Among various solutions, we are particularly interested in such solutions, for which the innermost part (core) is collapsing and approaching free-fall towards the center while simultaneously the outer part (envelope) is expanding and approaching a constant wind. The core-mass accretion rate is constant. The shock which connects the inner part and the outer part is propagating outward at a constant speed.

3 AXISYMMETRIC DISK CASES

For axisymmetric disk cases we introduce similarity transformation:

$$x = \frac{r}{at}\ ,\ \Sigma(r,\ t) = \frac{a\alpha(x)}{2\pi Gt}\ ,\ u(r,\ t) = av(x)\ ,\ C_A = aq(x)\ ,$$
$$M(r,\ t) = \frac{a^3t}{G}m(x)\ ,\ j(r,\ t) = \beta a^2 tm(x)\ ,\ \Phi(r,\ t) = a^2\phi(x)\ , \tag{3}$$

where

$$\phi(x) = -\frac{1}{2\pi x}\int_0^\infty \mathcal{X}(\xi/x)\alpha(\xi)\xi d\xi\ ,\ \mathcal{X}(X) = \oint \frac{d\psi}{(1+X^2-2X\cos\psi)^{1/2}}\ , \tag{4}$$

and α, v, q, m, ϕ are reduced quantities which are dimensionless functions of similarity independent-variable x and Σ, u, C_A, M, j, Φ are surface mass density, radial bulk velocity, Alfvénic speed, enclosed mass, specific angular momentum and gravitational potential respectively. The specific angular momentum is proportional to the enclosed mass because their ratio is assumed to be spatially uniform at the initial instant $t = 0$ and conserved during the successive evolution (Li & Shu 1997; Saigo & Hanawa 1998). Using these reduced variables we derive the ordinary differential equations (ODEs) as

$$[(x-v)^2-1-q^2]\frac{dv}{dx} = \left(f - \frac{\beta^2m^2}{x^3} - \frac{1}{x}\right)(x-v) + \frac{q^2}{2}\left(1-\frac{2v}{x}\right)\ , \tag{5}$$

$$[(x-v)^2-1-q^2]\frac{1}{\alpha}\frac{d\alpha}{dx} = f - \frac{\beta^2m^2}{x^3} - \frac{(x-v)^2}{x} + \frac{q^2}{2x}\frac{3x-4v}{x-v}\ , \tag{6}$$

$$[(x-v)^2-1-q^2]\frac{1}{q}\frac{dq}{dx} = \frac{f}{2} - \frac{\beta^2m^2}{2x^3} + \frac{q^2-2}{4(x-v)} + \frac{v}{2x}\frac{2-(x-v)^2}{x-v}\ , \tag{7}$$

where $f \equiv d\phi/dx$ together with the following auxiliary equations:

$$m = x\alpha(x-v)\ , \tag{8}$$

and equation (4). Equations (4)–(8) are the full set of ODEs to be solved. In particular, equations (5)–(7) are simultaneously solved for reduced density, reduced radial velocity and Alfvénic Mach number. In the monopole approximation, the reduced gravitational force is approximated as $f \sim m/x^2$.

There exists one exact solution for the ODE set, which is the rotational equilibrium state of a (magnetized) singular isothermal disk (MSID):

$$v = 0\ ,\ \alpha = \frac{1+D^2-Q^2/2}{x}\ ,\ f = \frac{1+D^2-Q^2/2}{x}\ ,\ m = (1+D^2-Q^2/2)x\ ,$$
$$\beta = \frac{D}{1+D^2-Q^2/2}\ ,\ q \equiv \text{constant} = Q\ . \tag{9}$$

Without magnetic fields ($q = 0$) and in the monopole approximation, we reduce the basic equation set (4)–(8) into

$$[(x-v)^2-1]\frac{dv}{dx} = \frac{\alpha(x-v)[1-\beta^2\alpha(x-v)]-1}{x}(x-v)\ ,$$
$$[(x-v)^2-1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \frac{\alpha[1-\beta^2\alpha(x-v)]-(x-v)}{x}(x-v)\ ,\ m = x\alpha(x-v)\ , \tag{10}$$

with the asymptotic solution when $x \to +\infty$ as

$$v \to V + \frac{1-A+\beta^2A^2}{x} + \frac{V(1-\beta^2A^2)}{2x^2} + \frac{2V^2+(A-1-\beta^2A^2)(A-4)}{6x^3}\ ,$$
$$\alpha \to \frac{A}{x} + \frac{A(1-A+\beta^2A^2)}{2x^3}\ ,\ m \to Ax\ . \tag{11}$$

For non-rotating disks we simply set $\beta = 0$ in the above equations.

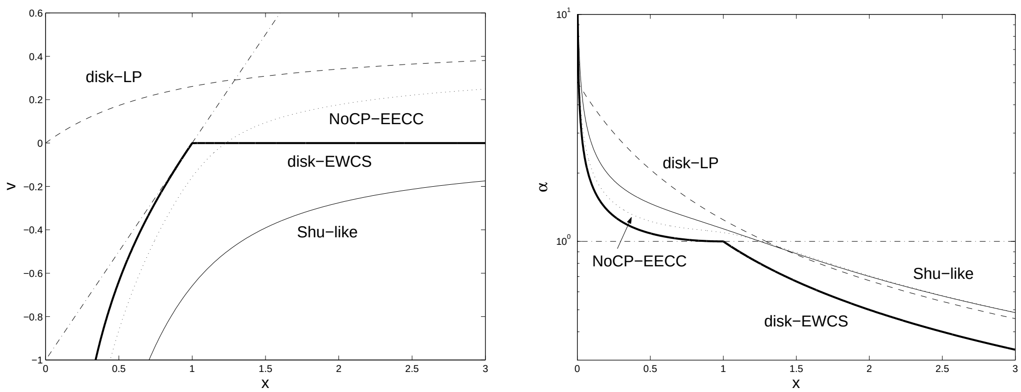

Fig. 1 Example solutions in non-rotating disks without magnetic fields. Left: Example solutions of $v(x)$ versus similarity variable x. The dash-dotted line is the sonic critical line $x - v = 1$. The light solid line is one of the Shu-like solutions and the heavy solid line is the disk-EWCS. The dotted line is one of the disk-EECC solutions which do not cross the sonic critical line. The dashed line is the disk-LP solution. Right: Example solutions of $\alpha(x)$ versus similarity variable x. Notations and line-types are the same as left.

3.1 Non-rotating Disks Without Magnetic Fields

For non-rotating $\beta = 0$ cases, we have asymptotic solutions when $x \to 0$ as either

$$v \to -\left(\frac{2m_0}{x}\right)^{1/2} , \ \alpha \to \left(\frac{m_0}{2x}\right)^{1/2} , \ m \to m_0 \ , \tag{12}$$

which stands for free-fall, or else

$$v \to \frac{x}{2} \ , \ \alpha \to B \ , \ m \to Bx^2 \ , \tag{13}$$

which stands for core-expansion. And the expansions of solutions near the sonic critical line $x - v = 1$ are

$$v = x_* - 1 + C_\mp (x - x_*) + ... \ , \qquad \alpha = 1 + (C_\mp - 1/x_*)(x - x_*) + ... \ , \tag{14}$$

where

$$C_\mp = \frac{1 \mp \sqrt{1 + 2(1/x_*^2 - 1/x_*)}}{2} \ , \tag{15}$$

with -minus and -plus signs denoting type 1 and type 2 solutions in reference to the spherical case (Hunter 1977; WS; LS04). Figure 1 shows several example solutions for the non-rotating case.

3.2 Rotating Disks Without Magnetic Fields

For rotating $\beta > 0$ cases, notice from equation (5) that the term of centrifugal force diverges as x^{-3} if $m \to m_0$, faster than other terms. So for rotating disks the material will not fall freely to the center as indicated by (12). Instead, we have asymptotic solutions when $x \to 0$ as either

$$\begin{aligned} v(x) &\to \frac{x}{2} - \frac{B}{12}x^2 + \frac{1}{32}\left[1 + \left(\frac{1}{3} + \beta^2\right)B^2\right]x^3 + ... \ , \\ \alpha(x) &\to B - \frac{B^2}{2}x + \frac{B}{8}\left[1 + \left(\frac{5}{3} + \beta^2\right)B^2\right]x^2 + ... \ ; \end{aligned} \tag{16}$$

or the quasi-equilibrium asymptotic solution

$$v(x) \to \frac{D^2-1}{(1+D^2)(1+D^2+\lambda)}\eta x^{\lambda+2}\ ,\ \alpha(x) \to \frac{1+D^2}{x} + \eta x^{\lambda}\ ,\ m(x) \to (1+D^2)x\ , \tag{17}$$

where

$$\lambda = \frac{-3+\sqrt{4D^2-3}}{2} > -1\ , \qquad D = \frac{1+\sqrt{1-4\beta^2}}{2\beta} > 1\ , \tag{18}$$

and η is a constant parameter. Asymptotic solution (17) is actually the quasi-equilibrium state with the leading order as the exact equilibrium SID solution and is rotating supersonically with rotational Mach number D exceeds unity.

By the isothermal shock jump condition (Tsai & Hsu 1995; Saigo & Hanawa 1998; Shu et al. 2002)

$$(v_d - x_s)(v_u - x_s) = 1\ , \qquad \frac{\alpha_u}{\alpha_d} = (v_d - x_s)^2\ , \tag{19}$$

we can connect the inner quasi-equilibrium solution to the outer envelope which approaches constant winds or inflows or breezes. In the above equations x_s is the shock location fixed in the similarity coordinates which suggests that the shock moves outward at constant speed $x_s a$, i.e., the physical radius of the shock discontinuity increases linearly with time.

4 DISCUSSION

The major difference of our self-similar solutions from those by other authors is that our solutions include not only those with constant inflow speeds or being quasi-static at large radii but also those with constant winds at large radii. Spatially, the novel solutions describe both collapsing part and expanding part, connected with or without shocks. Temporally, the fluid element at all radii is decelerated from expansion to collapse and the stagnation points as well as shock fronts propagate outward at constant speeds. The density and velocity profiles at large or small radii have well-defined power-law behaviors on radius, which thus can be used as useful input parameters in numerical radiation transfer codes. Another important aspect of these self-similar solutions is the core-mass accretion rate, which is constant. So the mass of the central object (either a proto-star or a black hole) increases linearly with time. We have extended the range of the core-mass accretion rate, which is only of specific values for several known self-similar solutions (i.e., for the EWCS, the core-mass accretion rate $m_0 = 0.975$).

The astrophysical applications of these self-similar solutions may appear in various circumstances: systems involving accretions and outflows. To be more specific, the evolution of young stellar objects (YSOs), H II regions around OB stars and planetary nebulae (PNe) may experience certain stages which can be described by these self-similar solutions (Shen & Lou 2004). By incorporating more realistic ingredients such as the polytropic equation of state, the toroidal environment and even general relativity, these solutions originally derived in isothermal spheres can be generalized to describe more complicated cases (e.g., core-collapse supernovae). In reference to our novel EECC solutions, we may propose the following scenario: the envelope at large radii is initially expanding due to external heating (i.e., from supernovae or evolved stars near star-forming regions); while the core region begins to collapse due to gravitational instabilities; the successive evolution of the core-envelope system may evolve into a self-similar state described by one of the EECC solutions. Such a situation may be applicable to the protostellar evolution where core accretion and large-scale outflows may concur. For planetary nebulae, it is interesting to note that in the classic interacting stellar wind (ISW) model, a much faster stellar wind from the core catches up with a slower dense wind–the remnant of the asymptotic giant branch phase–and together they form shocks; while our shocked EECC solutions imply that while the shocked expanding envelope is creating the observed planetary nebula, the core (i.e., the proto-white dwarf) is continuously accreting material. If it can accrete enough material to exceed the Chandrasekhar limit, there might be the possibility of igniting a Type Ia supernova explosion, without the requirement for a companion star!

References

Hunter C., 1977, ApJ, 218, 834
Hunter C., 1986, MNRAS, 223, 391
Krasnopolsky R., Königl A., 2002, ApJ, 580, 987
Landau L. D., Lifshitz E. M., 1959, In: Fluid Mechanics., New York: Pergamon Press,
Larson R. B., 1969, MNRAS, 145, 271
Li Z.-Y., Shu F. H., 1997, ApJ, 475, 237
Lou Y.-Q., Shen Y. 2004, MNRAS, 348, 717 (LS04)
Nakamura F., Hanawa T, Nakano, T., 1995, ApJ, 444, 770 (NHN)
Penston M. V., 1969, MNRAS, 144, 425
Saigo K., Hanawa T., 1998, ApJ, 493, 342
Sedov L. I., 1959, In: Similarity and Dimensional methods in Mechanics., New York: Academic Press,
Shen Y., Lou Y.-Q., 2004, ApJ, 611, L117
Shu F. H., 1977, ApJ, 214, 488
Shu F. H., Lizano S., Galli D., Cantó J., Laughlin G., 2002, ApJ, 580, 969
Tsai J. C., Hsu J. J. L., 1995, ApJ, 448, 774
Whitworth A., Summers D. 1985, MNRAS, 214, 1 (WS)