

# The Origin of Magnetars

## — The Role of Anisotropic neutron superfluid of Neutron Stars

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**Abstract** We estimate the strength of the induced magnetic field due to the Pauli paramagnetic moment of the  ${}^3\text{P}_2$  Cooper pairs for the anisotropic ( ${}^3\text{P}_2$ ) neutron superfluid under the applied magnetic field ( $B_0$ ) in neutron stars. The induced magnetic field of the anisotropic ( ${}^3\text{P}_2$ ) neutron superfluid is as follows.  $B^{(\text{in})} \approx \frac{1.9}{T_7} \eta B_0$  ( $T_7$  denotes the interior temperature of the neutron star in unit of  $10^7$  K),  $\eta = \frac{m({}^3\text{P}_2)}{0.1 M_\odot} R_{\text{NS},6}^{-3}$ . The induced magnetic field will gradually increase with the temperature of the neutron star decreasing in their late evolutionary stage. A magnetar may appear in a condition when  $T_7 \ll \eta$ . The upper limit of the magnetic field for the magnetars is  $B_{\text{max}}^{(\text{in})}({}^3\text{P}_2) \approx 2.02 \times 10^{14} \eta$ .

**Key words:** pulsars: general — neutron stars — magnetars

### 1 A QUESTION

Magnetic field for most of the neutron stars is in the range  $10^{10} < B$  (Gauss)  $< 10^{13}$  (e.g. Shapiro & Teukolski 1984). There probably are magnetars with ultra-strong magnetic field strength over the quantum critical threshold,  $H_{\text{cr}} = 4.414 \times 10^{13}$  Gauss (Duncan & Thompson 1992; Paczynski 1992; Usov 1992; Thompson & Duncan 1995, 1996). Anomalous X-ray Pulsars (AXPs) and Soft Gamma Repeaters (SGRs) are classes of candidates to magnetars (e.g. Kouveliotou et al. 1998, 1999; Hurley et al. 1999; Mereghetti & Stella 1995; Wilson et al. 1999). The magnetic field of the magnetars may be such strong as  $10^{14}$ – $10^{15}$  Gauss.

What is the origin of the ultra-strong magnetic field for the magnetars? It is a very interesting question. Some models for origin of magnetars have been proposed. Ferrario & Wickramasinghe (2005) suggest that the extra-strong magnetic field of the magnetars is descended from their stellar progenitor with high magnetic field core. Iwazaki (2005) proposed the huge magnetic field of the magnetars is some color ferromagnetism of quark matter. Recently Vink & Kuiper (2006) suggest that the magnetars originate from rapid rotating proto-neutron stars.

We are to propose a new model of origin of magnetars in this paper in terms of the behavior of the anisotropic  ${}^3\text{P}_2$  neutron superfluid.

### 2 ON ANISOTROPIC ${}^3\text{P}_2$ NEUTRON SUPERFLUID

There are two kinds of neutron superfluid interior of a neutron star in general: one is the isotropic  ${}^1\text{S}_0$  neutron superfluid with critical temperature of phase transition  $T_\lambda$ ,

$$T = T_\lambda({}^1\text{S}_0(n)) = \Delta({}^1\text{S}_0(n))/2k \approx 1 \times 10^{10} \text{ K}.$$

Here  $\Delta({}^1\text{S}_0(n))$  is the neutron pairing energy gap in the  ${}^1\text{S}_0$  state,  $k$  is the Boltzmann constant. The density range of the isotropic  ${}^1\text{S}_0$  superfluid neutron state is  $1 \times 10^{11} < \rho$  ( $\text{g cm}^{-3}$ )  $< 1.6 \times 10^{14}$ .

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Another superfluid is the anisotropic  ${}^3P_2$  neutron superfluid. Energy gap calculations show that the pairing gap varies with the density of neutrons (see fig. 8 of Elgagøy et al. (1996)) as follows. a) The  ${}^3PF_2$  neutron pair energy gap appears,  $\Delta({}^3PF_2(n)) > 0$ , in the region  $1.3 \times 10^{14} < \rho \text{ (g cm}^{-3}\text{)} < 7.2 \times 10^{14}$ , where  $\Delta({}^3PF_2(n))$  is an increasing function of the density for the region  $\rho < 3.3 \times 10^{14} \text{ g cm}^{-3}$  and it is a rapidly decreasing function of the density for the region  $\rho > 5.2 \times 10^{14} \text{ g cm}^{-3}$ . b) The maximum of the  ${}^3PF_2$  neutron pair energy gap is about 0.048 MeV at  $\rho \cong 4.2 \text{ g cm}^{-3}$ . c)  $\Delta({}^3PF_2(n))$  is almost a constant about the maximum with error less than 3% in the rather wide range  $3.3 \times 10^{14} < \rho \text{ (g cm}^{-3}\text{)} < 5.2 \times 10^{14}$ ;

The anisotropic  ${}^3PF_2$  superfluid neutron state can exist in the rather wide density region  $3.3 \times 10^{14} < \rho \text{ (g cm}^{-3}\text{)} < 5.2 \times 10^{14}$ , when the temperature decreases down to the critical temperature,  $T_\lambda({}^3PF_2(n))$ , which corresponds to the maximum of the  ${}^3PF_2$  neutron pair energy gap

$$T \leq T_\lambda({}^3PF_2(n)) = \Delta_{\max}({}^3PF_2(n))/2k \approx 2.78 \times 10^8 \text{K}. \quad (1)$$

It should be pointed out that the neutron fluid is still in the normal Fermi fluid state in the region  $1.6 \times 10^{14} < \rho \text{ (g cm}^{-3}\text{)} < 3.3 \times 10^{14}$  and  $\rho > 5.2 \times 10^{14} \text{ g cm}^{-3}$  when the temperature is near the maximum transition temperature  $T_\lambda({}^3PF_2(n))$ . However, the anisotropic ( ${}^3PF_2(n)$ ) superfluid region will gradually enlarge when the temperature steadily decreases further, because the neutron (normal) Fermi fluid in some region will be transformed to the anisotropic ( ${}^3PF_2$ ) neutron superfluid when its temperature further decreases below the corresponding transition temperature,  $T \leq \Delta({}^3PF_2(n))/2k$ .

A  ${}^3P_2$  neutron Cooper pair has a spin angular momentum with a spin quantum number,  $\sigma = 1$ . The  ${}^3P_2$  neutron Cooper pair has also a magnetic moment. The magnetic moment of the  ${}^3P_2$  neutron Cooper pair is twice that of the abnormal magnetic moment of a neutron,  $2\mu_n$  on magnitude, and its projection on an external magnetic field ( $z$ -direction) is  $\mu_{\text{pair}} = -\sigma_z \times (2\mu_n)$ ,  $\sigma_z = 1, 0, -1$ .  $\mu_n = 0.966 \times 10^{-23} \text{ erg Gauss}^{-1}$ .

Behavior of the  ${}^3P_2$  neutron superfluid is similar to behavior of the liquid  ${}^3\text{He}$  in very low temperature (Leggett 1975):

- (1) The projection distribution for the magnetic moment of the  ${}^3P_2$  neutron Cooper pairs in a circumstance without external magnetic field is on equal probability, or Equal Spin Pair (ESP) phase similar to the A-phase of the liquid  ${}^3\text{He}$  in very low temperature (Leggett 1975). The  ${}^3P_2$  neutron superfluid is basically isotropic without significant magnetic moment in the circumstance without external magnetic field. We name it as the A- phase of the  ${}^3P_2$  neutron superfluid.
- (2) However, The projection distribution for the magnetic moment of the  ${}^3P_2$  neutron Cooper pairs in the circumstance with external magnetic field is not on equal probability. The number of the  ${}^3P_2$  neutron Cooper pair with paramagnetic magnetic moment is more than one with diamagnetic magnetic moment. Therefore, the  ${}^3P_2$  neutron superfluid have a whole induced paramagnetic magnetic moment and its behavior is anisotropic in the circumstance with external magnetic field. We name it as the B- phase of the  ${}^3P_2$  neutron superfluid similar to the B-phase of the liquid  ${}^3\text{He}$  in very low temperature (Leggett 1975).

### 3 INDUCED PARAMAGNETIC MAGNETIC MOMENT OF THE ${}^3P_2$ NEUTRON SUPERFLUID IN THE B-PHASE

The induced paramagnetic magnetic moment of the  ${}^3P_2$  neutron superfluid in the B-phase may be simply estimated as follows:

The system of the  ${}^3P_2$  neutron Cooper pairs with spin quantum,  $s = 1$ , may be taken as a Bose-Einstein system. All  ${}^3P_2$  neutron Cooper pairs may condense into same state with lowest energy (the fundamental state,  $E = 0$ ) in very low temperature. However, a paramagnetic magnetic moment possesses lower energy than a diamagnetic magnetic moment under an external magnetic field by stability. Therefore, the  ${}^3P_2$  neutron Cooper pair has energy  $\sigma_z \times 2\mu_n B_0$  ( $\sigma_z = 1, 0, -1$ ) under the magnetic field due to its magnetic moment  $[-\sigma_z \times 2\mu_n]$ , where  $B_0$  is the applied magnetic field the background magnetic field in existence before appearance of the  ${}^3P_2$  neutron superfluid. We denote the number of  ${}^3P_2$  neutron Cooper pairs with spin projection  $\sigma_z = 1, 0, -1$  by  $n_1, n_0$  and  $n_{-1}$ , respectively. Their relative ratios are

$$\frac{n_{-1}}{n_0} = e^{2\mu_n B_0/kT}, \quad \frac{n_{+1}}{n_0} = e^{-2\mu_n B_0/kT}, \quad (2)$$

$$n_{-1} + n_0 + n_{+1} = n_n(^3P_2). \quad (3)$$

The difference of the  $^3P_2$  neutron Cooper pair number with paramagnetic and diamagnetic magnetic moment is

$$\Delta n_{\mp} = n_{-1} - n_{+1} = n_n f\left(\frac{\mu_n B_0}{kT}\right), \quad (4)$$

$$f(x) = \frac{2 \sinh(2x)}{1 + 2 \cosh(2x)}. \quad (5)$$

The temperature factor  $f(\mu_n B_0/kT)$  is introduced for taking into account the effect of thermal motion on the direction of the magnetization (Thermal motion will cause the magnetic moments of the  $^3P_2$  Cooper pairs to point at randomness as soon as possible). It is an increasing function:

$$f(x) \approx 4x/3, \quad x \ll 1, \quad (6)$$

$$f(x) \rightarrow 1 \quad \text{when } x \gg 1. \quad (7)$$

Neutrons combined into the  $^3P_2$  Cooper pairs are just in a thin layer with thickness  $\sqrt{2m_n \Delta_n(^3P_2)}$  near the surface of the Fermi sphere in the momentum sphere. The fraction of neutrons combined into the  $^3P_2$  Cooper pairs is

$$q = \frac{4\pi p_F^2(n) [2m_n \Delta(^3P_2(n))]^{1/2}}{(4\pi/3)p_F^3} = 3 \left[ \frac{\Delta(^3P_2(n))}{E_F(n)} \right]^{1/2}. \quad (8)$$

The Fermi energy of the neutron system may be calculated by the formula

$$E_F = \frac{1}{2m_n} \left(\frac{3}{8\pi}\right)^{2/3} h^2 N_A^{2/3} (Y_n \rho)^{2/3} \approx 60 \left(\frac{\rho}{\rho_{\text{nuc}}}\right)^{2/3} \text{ MeV}. \quad (9)$$

(We use the relations  $E_F(n) = p_F^2(n)/2m_n$ ,  $p_F(n) = (3/8\pi)^{1/3} h n_n^{1/3}$ ,  $n_n \approx N_A \rho$ ).

The energy gap of the anisotropic neutron superfluid is  $\Delta(^3P_2) \sim 0.05 \text{ MeV}$  (Elgagøy et al. (1996)),  $q \sim 8.7\%$ . Thus, the total number of neutrons into the  $^3P_2$  Cooper pairs is

$$N_n(\text{pair}) \approx q N_A m(^3P_2(n))/2. \quad (10)$$

Therefore, the total difference of the  $^3P_2$  neutron Cooper pair number with paramagnetic and diamagnetic magnetic moment is

$$\Delta N_{\mp} = N(^3P_2) f\left(\frac{\mu_n B_0}{kT}\right) = \frac{q}{2} N_A m(^3P_2) f\left(\frac{\mu_n B_0}{kT}\right). \quad (11)$$

The total induced magnetic moment, of the anisotropic neutron superfluid is thus

$$\mu_{\text{pair}}^{(\text{tot})}(^3P_2) = 2\mu_n \times \Delta N_{\mp} = \mu_n q N_A m(^3P_2) f(\mu_n B_0/kT), \quad (12)$$

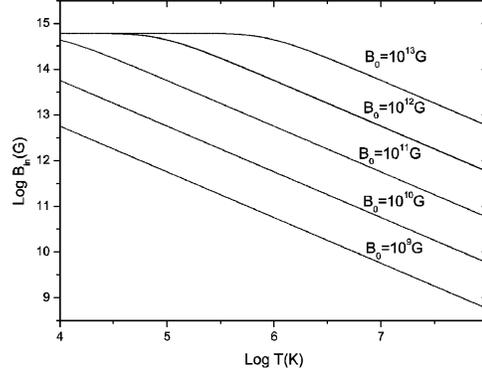
where  $m(^3P_2)$  is the mass of the anisotropic neutron superfluid in the neutron star,  $N_A$  is the Avogadro constant. According to the formulae of the magnetic moment with the polar magnetic field  $|\mu_{\text{NS}}| = B_p R_{\text{NS}}^3/2$  (Shapiro and Teukolski 1984, here  $B_p$  is the polar magnetic field strength and  $R_{\text{NS}}$  is the radius of the neutron star), the induced magnetic field by the total magnetic moment of the anisotropic neutron superfluid is then

$$B^{(\text{in})} = \frac{2\mu_{\text{pair}}^{(\text{tot})}(^3P_2)}{R_{\text{NS}}^3} = \frac{2\mu_n q N_A m(^3P_2)}{R_{\text{NS}}^3} f(\mu_n B_0/kT). \quad (13)$$

Given the background magnetic field ( $B_0$ ), the induced magnetic field for the anisotropic neutron superfluid increases with decreasing of the temperature which is shown in the Figure 1.

We have  $\mu_n B_0 \ll kT$ , for the general case  $B_0 < 10^{13}$  Gauss and  $T > 10^7$  K and

$$B^{(\text{in})} \approx \frac{8}{3} \frac{\mu_n q N_A m(^3P_2)}{R_{\text{NS}}^3} \frac{\mu_n B_0}{kT}, \quad (14)$$



**Fig. 1** The curves show the variation of the induced magnetic field,  $B_{in}$ , of the  ${}^3\text{P}_2$  pairs for the anisotropic neutron fluid with the temperature interior the neutron star for different initial background magnetic field,  $B_0$ .  $T_7 = T/10^7$  K.

or

$$B^{(in)} \approx \frac{1.9}{T_7} \eta B_0, \quad (15)$$

$$\eta = \frac{m({}^3\text{P}_2)}{0.1 M_\odot} R_{NS,6}^{-3}. \quad (16)$$

The induced magnetic field,  $B^{(in)}$ , of the anisotropic neutron superfluid will increase with the interior temperature decreasing and it will exceed the initial background magnetic field in existence before appearance of the anisotropic neutron superfluid when the temperature decreases down to  $T_7 < 1.9\eta$ .  $B^{(in)}$  will become much stronger than the previous background magnetic field when the temperature of the neutron star decreases down to such as  $T_7 \ll \eta$ . Its magnetic field can be much more than the Landau critical value ( $4.414 \times 10^{13}$  Gauss). The neutron star will become a magnetar.

Thus we may get a very important conclusion: the ultra-strong magnetic field of the magnetar is really the induced magnetic field by the paramagnetic moments of  ${}^3\text{P}_2$  Cooper pair of the anisotropic neutron superfluid in very low temperature.

#### 4 THE UPPER LIMIT OF THE MAGNETIC FIELD OF THE MAGNETARS

The Formulae (14)–(15) are valid only in the condition  $\mu_n B^{(tot)} \gg kT$ . The corresponding formulae should be calculated from Equation (13) with Equation (5) when the temperature decreases down to  $T \sim \mu_n B^{(in)}/kT$ . The temperature factor  $f(\mu_n B_0/kT)$  tends to 1 when the temperature decreases down to the absolute zero of Kelvin. Actually,  $f(\mu_n B_0/kT) \sim 1$  as long as  $\mu_n B_0/kT \gg 1$ . For an example, this is true when  $T < 10^4$  K if  $B_0 = 10^{12}$  Gauss.

There is an upper limit of the induced magnetic field of  ${}^3\text{P}_2$  superfluid according to the Eq. (13) due to the maximum of the temperature factor  $f(\mu_n B_0/kT)$  is 1. In physics, the upper limit will be arrived when all magnetic moments of  ${}^3\text{P}_2$  neutron Cooper pairs are arranged with paramagnetic direction as the temperature decreases down to the absolute zero of Kelvin. The upper limit of the induced magnetic moment of  ${}^3\text{P}_2$  superfluid is

$$\mu_{tot}^{(up)}({}^3\text{P}_2) = qN_A m({}^3\text{P}_2(n))\mu_n \approx 1.0 \times 10^{32} \left[ \frac{m({}^3\text{P}_2)}{0.1 M_\odot} \right]. \quad (\text{c.g.s.}) \quad (17)$$

The upper limit of the magnetic field for the magnetars is

$$B_{max}^{(in)}({}^3\text{P}_2) = \frac{2\mu_n q N_A m({}^3\text{P}_2)}{R_{NS}^3} \text{ Gauss} \approx 2.02 \times 10^{14} \eta \text{ Gauss}. \quad (18)$$

The maximum of the magnetic field for the magnetar depends the total mass of the anisotropic neutron superfluid of the neutron star. It is well known that the upper limit of the mass for the neutron stars is more than  $2.0 M_{\odot}$ . It is possible that the mass of the anisotropic neutron superfluid of the heaviest neutron star may be about  $(1-1.5) M_{\odot}$ . Hence, the maximum of the magnetic field for the heaviest magnetar may arrive at  $(2-3) \times 10^{15}$  Gauss.

## 5 CONCLUSIONS

The conclusion of this paper is that the ultra magnetic field of the magnetars originates really by the induced magnetic field due to the induced paramagnetic moment of  ${}^3\text{P}_2$  superfluid with significant mass more than  $0.1 M_{\odot}$  in the temperature much lower than  $1 \times 10^7$  K.

It should be pointed out that the neutron fluid is still in the normal Fermi fluid state in the region  $1.6 \times 10^{14} < \rho$  ( $\text{g cm}^{-3}$ )  $< 3.3 \times 10^{14}$  and  $\rho > 5.2 \times 10^{14} \text{ g cm}^{-3}$  when the temperature is near the maximum transition temperature  $T_{\lambda}({}^3\text{P}_2(n))$ . However, the anisotropic ( ${}^3\text{P}_2(n)$ ) superfluid region will gradually enlarge (to see figure 8 of Elgagøy et al. 1996), when the temperature steadily decreases further, because the neutron (normal) Fermi fluid in some region will be transformed to the anisotropic ( ${}^3\text{P}_2$ ) neutron superfluid when its temperature further decreases below the corresponding transition temperature,  $T \leq \Delta({}^3\text{P}_2(n))/2k$ . Thus both the mass of the anisotropic ( ${}^3\text{P}_2$ ) neutron superfluid and the parameter,  $\eta$ , will increase with temperature decreasing.

For all neutron stars with a significant  ${}^3\text{P}_2$  superfluid, their magnetic field will increase gradually as the star is cooling continually and all of them will evolution towards magnetars.

## 6 PULSAR'S SPIN DOWN AND AGE OF PULSARS

We will now consider the process of pulsar spin down and discuss the age of pulsars in the framework of our formalism.

Magnetic field may be calculated in terms of the standard magnetic dipole model

$$P\dot{P} = A_0 B_{p,12}^2, \quad A_0 = 2.44 \times 10^{-16} \sin^2 \alpha \quad (19)$$

or

$$B_{p,12} \approx 3.3 \times 10^7 \sqrt{P\dot{P}} \text{ Gauss}. \quad (20)$$

The age of a pulsar may be calculated by the formulae

$$t = \int_{P_0}^P \frac{P dP}{A_0 B_{p,12}^2} = \frac{1}{2P\dot{P}} (P^2 - P_0^2), \quad \frac{t}{\tau} = 1 - \left(\frac{P}{P_0}\right)^2, \quad (21)$$

where  $\tau = P/2\dot{P}$  is the characteristic age of the pulsar,  $P_0$  is the initial period of the pulsar.

Taking the contribution of neutrino radiation by the superfluid neutron vortices into account in our hybrid model for pulsar spin down (Peng, Huang & Huang 1982), we modify Equation (19) as follows

$$P\dot{P} = A_1 (B_{p,12}^2 + B_1 G(n) P^3), \quad G(n) = \frac{\bar{n}^7}{\bar{n}}, \quad B_1 = 3.01 \times 10^{-8} \sin^{-2} \alpha, \quad (22)$$

where  $n$  is the vortex quantum number of both the isotropic and the anisotropic superfluid neutron vortices, which decreases with the increasing period of the pulsar. We may assume that

$$G(n) = G(n_0) \left(\frac{P}{P_0}\right)^{-\beta}, \quad 0 < \beta \leq 1 \quad (23)$$

The age of the pulsar is calculated by

$$t = \int_{P_0}^{P_1} \frac{P}{A_1 (B_{12}^{(0)})^2 + B_1 G(n_0) P_0^\beta P^{3-\beta}} dP + \int_{P_1}^P \frac{P}{A_1 (B_{12}^2(t) + B_1 G(n_0) P_0^\beta P^{3-\beta})} dP, \quad (24)$$

where  $P_1$  is the pulsar period when the interior temperature of the neutron star decreases down to  $\eta$  (while the induced magnetic field exceeds the initial background one).

It is well known that the superfluid neutron vortices are energetically more favorable in the ground state of  $n=1$  in the thermo-dynamical equilibrium of the superfluid vortices. However, the superfluid vortices in the young pulsars are in a highly non-equilibrium state. This is because a significant fraction of the angular momentum of the rotational pre-supernova is reserved in the turbulent classical vortices of the nascent neutron star during supernova explosion. Therefore, the initial quantum number  $n_0$  of these quantized vortices (for both  $^1S_0$  and  $^3PF_2$  superfluid neutron vortices) are very large after the phase transitions just mentioned. For typical superfluid neutron vortices, ( $J_{NSV}(^3PF_2) > 10^{-10} - 10^{-8} J(^3PF_2)$ ), we have  $n_0 > 10^3 - 10^5$  (Peng, Luo & Chou 2006). Combining with Equation (22), it is easily seen that the term  $(B_{12}^{(0)})^2$  in the first integral of Equation (24) may be neglected. Since the initial magnetic field strength being not very strong, the spin down rate of the pulsar was dominated by the neutrino radiation of the superfluid neutron vortices (the second term in the Eq. (22)) in the early stage of the pulsar when its interior temperature was higher than  $2.78 \times 10^8$  K.

However, after the presence of the ( $^3PF_2$ ) neutron superfluid and appearance of the induced strong magnetic field, the spin down rate of the pulsar (Eq. (22)) is contributed by both the magnetic dipole radiation and the neutrino radiation of the superfluid neutron vortices.

**Acknowledgements** The author is very grateful to Professor Chich-gang Chou for his help of improving English of the paper and grateful to Dr. Xin-Liang Luo, Hua Bai for their help in preparing the paper. This research is supported by Chinese National Science Foundation Nos. 10573011 and 10273006, and the Doctoral Program Foundation of State Education Commission of China.

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