The Energetic Problem of X-Ray Emission from the Coma Cluster

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Abstract The stochastic acceleration of subrelativistic electrons from a background plasma is studied to find an explanation of the hard X-ray emission detected from the Coma cluster. We show that the temporal evolution of the electron distribution functions has, at its final stationary stage, a rather specific form determined by the interactions with charged background particles and electromagnetic fluctuations. These distribution functions cannot be described by simple exponential or power-law expressions. A broad transfer region is formed by Coulomb collisions at energies between the Maxwellian and power-law parts of the distribution functions. In this region the radiative lifetime of a single quasi-thermal electron differs greatly from the lifetime of the distribution function as a whole. This solves the problem of rapid cluster overheating by nonthermal electrons (Petrosian, 2001): while Petrosian’s estimates are correct for nonthermal particles they are inapplicable in the quasi-thermal range.

Key words: galaxy clusters – X-rays: variables: in-situ particle acceleration

1 INTRODUCTION

The problem of the energetics of the emitting electrons in clusters of galaxies is one of the key. First, we recall Petrosian’s arguments (see Petrosian, 2001). He estimated the yield of bremsstrahlung photons as \( Y \sim \langle dE/dt \rangle_{br}/\langle dE/dt \rangle_i \sim 3 \times 10^{-6} \) in the energy range of 20–80 keV. Here \( \langle dE/dt \rangle_i/(dE/dt)_{br} \) is the ratio of ionization to bremsstrahlung losses. Then for the X-ray flux from Coma in this energy range, \( F_X \sim 4 \times 10^{43} \text{ erg s}^{-1} \), a large amount of energy \( F_c \sim F_X/Y \sim 10^{49} \text{ erg s}^{-1} \) is transmitted from the accelerated electrons to the background plasma by ionization losses. As a result the intracluster plasma temperature should rise to \( > 10^8 \) K in a time of only \( 3 \times 10^7 \) years. These conclusions were obtained under the assumption that the lifetime of a single electron equals the lifetime of the particle distribution function. These estimates are correct only in the case that the electrons are nonthermal and therefore collisionless. However, they cannot be used in energy ranges where the spectrum is formed by Coulomb collisions because the lifetime of particles differs strongly from that of the distribution function.

In this respect, I would like to clarify the remark given after my talk. It was mentioned that the origin of emitting electrons is not very serious because they can easily be produced as secondaries (from \( \pi^\pm \)-decay or as knock-on electrons) by primary protons. To my opinion, this is a misunderstanding of the problem. The problem is not to produce electrons but how to produce necessary power which according to Petrosian’s estimates is huge if the emitting particles are nonthermal independently of that they are primary or secondary. In the case of secondary knock-on electrons the problem of power is only aggravated because they are definitely nonthermal and only a part of the energy of primary protons is transformed into the energy of high energy electrons.

2 THE COEFFICIENT OF IN-SITU ACCELERATION

The evolution of the distribution function, \( f(p, \mu, z, t) \), of particles which are scattered by electromagnetic fluctuations is described by the well-known Fokker-Planck equation

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\[ \frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu} \left( D_{\mu\mu} \frac{\partial f}{\partial \mu} \right) + \frac{\partial}{\partial \mu} \left( D_{\mu p} \frac{\partial f}{\partial p} \right) + \frac{\partial}{\partial p} \left( D_{p\mu} \frac{\partial f}{\partial \mu} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 D_{pp} \frac{\partial f}{\partial p} \right) + Q_0, \]

where \( v \) and \( p \) are the particle velocity and momentum, \( \mu \) is the cosine of its pitch-angle and \( Q_0 \) is the source function. Charged particle scattering is classically interpreted as particle resonant interaction with electromagnetic fluctuations in the intracluster medium. In order to calculate the coefficient \( D_{ij} \), one should sum over all resonances. Steinacker & Miller (1992) and Miller & Steinacker (1992) showed that the gyroresonant acceleration of electrons from thermal to relativistic energies requires a very broad spectrum of waves.

For a power-law spectrum of magnetic fluctuations \( W(k) \) which is a function of the wave-number \( k \)

\[ W(k) = \frac{A}{\Omega H} k^{-\beta}, \]

where \( A \) is the normalization constant and

\[ \Omega_H = \frac{eH_0}{mc}, \quad \omega_H = \frac{ZeH_0c}{E_{\text{tot}}}, \]

the momentum diffusion coefficient describing electron in-situ acceleration has the form

\[ D_p(p) = \frac{1}{2} \int_{-1}^{1} d\mu D_{pp}(\mu). \]

Here \( \mu \) is the cosine of particle pitch-angle, \( \beta = v/c \),

\[ \frac{D_{pp}}{\omega_H(mc)^2} \approx \frac{\pi A}{3a} \left( \frac{\beta |\mu|}{a} \right)^{\frac{j-1}{4}} (1 - \mu^2), \]

\[ a = \omega_p^2/\omega_H^2 \]

and

\[ \omega_p^2 = \frac{4\pi ne^2}{m_e} + \frac{4\pi n^2}{m_p}, \]

where \( m_e \) and \( m_p \) are the electron and proton rest masses.

3 SPECTRUM OF ACCELERATED ELECTRONS IN THE HALO OF THE COMA CLUSTER

For the Coma cluster we take the parameters presented in Liang et al. (2002): the best estimates of the intracluster temperature are \( T = 8.21 \pm 0.09 \) keV from Ginga. An excess of hard X-rays above the thermal component has been detected in the energy range 20–80 keV (Fusco-Femiano, 1999). As in Liang et al. (2002), we estimate the average electron density within the Beppo-SAX field of view of 1.3 (i.e. a volume of \( V \sim 1.7 \times 10^{75} \) cm\(^3\)) to be \( n_e \sim 1.23 \times 10^{-4} \) cm\(^{-3}\).

Using Eqs. (2), (4) and (5), we can calculate the coefficient of momentum diffusion for any particle energy. Following Dogiel (2000) and Liang et al. (2002), we use a kinetic equation, which is valid for subrelativistic and relativistic energies, to describe particle in-situ acceleration from the background plasma including the influence of Coulomb collisions and stochastic acceleration

\[ \frac{\partial f}{\partial \tilde{t}} - \frac{1}{p^2} \frac{\partial}{\partial p} \left( A(p) \frac{\partial f}{\partial p} + B(p)f \right) = 0. \]

This equation is written in the dimensionless variables \( \tilde{p} \) and the dimensionless time \( \tilde{t} = t\nu \) and diffusion coefficients \( \tilde{D}_p(p) = D_p(p)/(\nu mkT) \). The frequency \( \nu \) is

\[ \nu = \frac{2\pi ne^2\nu^2 m}{\sqrt{mkT}}. \]
For notational simplicity, we drop the tilde in Eq. (7) and hereafter. Here

$$B(p) = p^2 \left( \frac{dp}{dt} \right)_i,$$

and

$$A(p) = B(p) \frac{\gamma}{\sqrt{\gamma^2 - 1}} \sqrt{\frac{kT}{mc^2}} + p^2 D_p(p)$$

(10)

where $r_e = e^2/(mc^2)$ is the classical electron radius.

The dimensionless rate of ionization loss has the form

$$\left( \frac{dp}{dt} \right)_i = \frac{1}{p} \sqrt{p^2 + \frac{mc^2}{kT} \frac{\gamma}{\sqrt{\gamma^2 - 1}}}$$

$$\times \left\{ \ln \left[ \frac{Emc^2(\gamma^2 - 1)}{\hbar^2 \omega_p^2 \gamma^2} \right] + 0.43 \right\},$$

(11)

where $\omega_p$ is plasma frequency and $E(p)$ is particle kinetic energy.

The quasi-steady state solution of Eq. (7), obtained by Gurevich (1960), is

$$f = \sqrt{\frac{2}{\pi}} n(t) \exp \left( - \int_0^p \frac{B(v)}{A(v)} dv \right) G(p),$$

(12)

where

$$G(p) = \frac{\int [dv/A(v)] \exp \left( \int_0^p \frac{[B(t)/A(t)] dt}{p} \right)}{\int_0^\infty [dv/A(v)] \exp \left( \int_0^p [B(t)/A(t)] dt \right)},$$

(13)

and $n(t)$ is the density of background plasma.

The stochastic acceleration violates the equilibrium condition and as a result Coulomb collisions form a flux of particles running-away into the acceleration region in the momentum range $p > p_M$ where the value of $p_M$ is determined by parameters of acceleration.

The acceleration forms the collisionless part of the spectrum in the momentum range $p > p_{\text{inj}} > p_M$ where the injection momentum $p_{\text{inj}}$ is determined from the condition

$$\left( \frac{dp}{dt} \right)_i \sim pD_p(p).$$

(14)

### 4 POWER OF ELECTRONS NECESSARY FOR BREMSSTRAHLUNG X-RAYS

In the range $p > p_M$ there are two excesses above the equilibrium Maxwellian spectrum are formed. When $p_M < p < p_{\text{inj}}$, the excess is formed by Coulomb collisions (the collisional regime of quasi-thermal particles), and one can imagine the spectrum there as a distorted Maxwellian function. For $p > p_{\text{inj}}$ the spectrum is formed by particle interactions with plasma waves (the collisionless regime of nonthermal particles). Petrosian’s arguments can be applied to the range of $p > p_{\text{inj}}$ only.

We estimate the power necessary to support this excess above the equilibrium state at any energy $E_e$. Then from Eq. (1) we obtain an expression for the rate of change of the energy content of the electrons $W_e$

$$\frac{\partial W_e}{\partial t} = \Phi$$

$$= 4\pi V E_e \left( \frac{1}{p} \frac{\partial f}{\partial p} + p^2 \left( \frac{dp}{dt} \right)_i \frac{\partial f}{\partial p} + f \right),$$

(15)
where the total number of particles with momentum $\geq p$ in a volume $V$, $F_e(p) = V \int_p^\infty f(p)4\pi p^2 dp$, and the total electron energy in this volume, $W_e \sim E_e F_e$. As follows from this equation, Petrosian’s estimates of the power are completely correct for the energy range of nonthermal particles ($p > p_{\text{inj}}$). Indeed, if the energy interval of 20–80 keV is in the range $p > p_{\text{inj}}$, then $Y \sim 3 \times 10^{-6}$ and $F_e \sim F_X/Y \sim 10^{49} \text{ erg s}^{-1}$.

However, for the temperature of the Coma intracluster gas with the temperature $T = 8.21 \text{ keV}$, the electrons emitting excess at 20–80 keV are just quasi-thermal. Therefore the heating they require is almost two orders of magnitude below Petrosian’s estimate. This resolves the energetic problem raised by Petrosian: the ionization losses of the radiating electrons heat the plasma relatively slowly, so that a temperature $> 10^8 \text{ K}$ will be reached only after $> 10^{9}$ years.

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References

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